Modified Multivariate Control Chart Using Spatial Signs and Ranks for Monitoring Process Mean: A Case of t-Distribution

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Abstract

This paper purposes to modify multivariate control chart with multivariate spatial signs and ranks. The Monte Carlo simulation is used to compare the performance of the control chart based on the ARL. The modified control chart is sensitive to small shifts monitoring in the process mean vector and provide fast signals for detecting small shifts with more efficiency for symmetric t-distribution data. The results show that the SSRM outperforms MEWMA, dMEWMA, and SSRdM control charts in detecting small shifts of all smoothing parameter and in detecting moderate and large shifts outperform with the large smoothing parameter. The most industrial has current data and the industrial situations prefer large values of the smoothing parameter, thus the SSRM is more appropriate to kurtosis data in detecting small shifts for process mean monitoring and not interest in the resolution of data.

Keywords
Average Run Length, MEWMA, dMEWMA, Control Chart, Spatial Signed-Rank

1. Introduction

Quality is generally determined by several quality characteristics which may be correlated. Each of these quality characteristics must satisfy certain specifications. The quality of the product depends on the combined effect of many input variables rather than their individual values. In monitoring situations, multivariate control charts are powerful tools. This method considers the correlation between variables, monitors more than one variable simultaneously and takes this correlation of process variables into account by the mean vector. The multivariate control charts reflect the process situation more precisely and are able to detect the out-of-control situation. Then it is needed for monitoring and diagnosis purposes in modern manufacturing systems. However, the non-normality is a factor that can reduce the effectiveness of the multivariate control charts (Rama-Mohana-Rao 2013).

The main advantage of the non-parametric control charts is the flexibility derived from not needing to assume any parametric probability distribution for the underlying process, at least as far as establishing and implementing control charts are concerned. Obviously, this is very beneficial in the field of process control, particularly in the start-up situation where not much data is available to use a parametric procedure (Das 2008).

This research interests in the modification of Multivariate Control Chart Using Spatial Signs and Ranks for monitoring process mean by applying with t-distribution in the simulation.

2. Multivariate Spatial Signs and Ranks

Oja (2010) lets $X = (x_1, x_2, \ldots, x_n)'$, $i = 1, 2, \ldots, n$ be an nxp dataset. The multivariate concepts of spatial signs, spatial ranks and spatial signed-rank are then given in the following. The empirical spatial signs, spatial ranks and spatial signed-rank functions $U(x)$, $R(x) = R_x(x)$, and $Q(x) = Q_x(x)$ are defined as

$$U(x) = \begin{cases} \|x\|^{-1}x, & x \neq 0 \\ 0, & x = 0 \end{cases} \tag{1}$$

$$R(x) = AVE\{U(x - x_i)\} \quad (2)$$

$$Q(x) = ½[R_x(x) + R_{-x}(x)] \quad (3)$$

where $||x|| = (x'x)^{1/2}$ is the Euclidean length of vector x.

Observe that in the univariate case regular signs, ranks, and signed-rank functions are obtained. Clearly multivariate signed-rank function Q(x) is also odd; that is, $Q(-x) = -Q(x)$. The observed spatial signs are $U_i = U(x)$. As in the univariate case, the observed spatial ranks are certain averages of signs of pairwise differences

$$R_i = R(x_i) = AVE_j\{U(x_i - x_j)\} \quad (4)$$

Finally, the observed spatial signed-rank are given as

$$Q_i = Q(x) = ½\ AVE_j\{U(x_i - x_j) + U(x_i + x_j)\} \quad (5)$$

The spatial signs U_i is just a direction vector of length one (lying on the unit p-sphere) whenever $x_i \neq 0$. The centered ranks R_i and signed-rank Q_i lie in the unit p-ball. The direction of $R_i(Q_i)$ roughly tells the direction of y_i from the center of the data cloud (the origin), and its length roughly tells how far away this point is from the center (the origin). The next theorem collects some equivariant properties.

The spatial signed-rank of the transformed observations, $\tilde{Q}_i$ are called standardized spatial signed-rank. The multivariate spatial signed-rank test based on the inner standardization then rejects H_0 for large values of

$$Q^2(XS^{-1/2}) = 1_n' \tilde{Q} (\tilde{Q}' \tilde{Q})^{-1} \tilde{Q}' 1_n = np.|AVE\{\tilde{Q}_i\}|^2/AVE\{|\tilde{Q}_i|^2\} \quad (6)$$

which is simply nxp times the ratio of the squared length of the average signed-rank to the average of squared length of signed-rank. Then the multivariate spatial signed-rank test statistic calculated for the transformed data set, $Q^2(XS^{-1/2})$ is affine invariant and under the null hypothesis $H_0 : \mu = 0$

In this research, the R package, named "SpatialNP" by Sirkiä et al. (2017), is used to transform randomly generated data set to be spatial signed-rank data. And the R package, named "MNM" by Nordhausen et al. (2018), is used to test statistical of spatial signed-rank dataset for the affine invariant test.

The methods based on spatial signed-rank are more robust, more efficient for heavy-tailed distributions than normal theory based methods (Möttönen and Tienari 1997). Chakraborty and Chaudhuri (1998) presented the spatial signs and ranks methods can be developed by using the well-established transformation-retransformation techniques with an inner standardization. This computationally intensive modifications of the transformation-retransformation procedures can be used as general tools to achieve affine invariant and equivariant properties as Nevalainen et al. (2018) reported.

3. Modified Multivariate Control Chart

3.1 SSRM Control Chart

The Spatial Signed-Rank Multivariate Exponentially Weighted Moving Average (SSRM) control chart is modified from Multivariate Exponentially Weighted Moving Average (MEWMA) control chart. The equation of MEWMA is $y_i = \Lambda x_i + (I-\Lambda)y_{i-1}$, i=1, 2,…,n (Alkahtani and Schaffer, 2012)

where y_i is the i^{th} MEWMA

x_i is the i^{th} observation vector of $t_p(\upsilon=5)(\mu_{xi}, \Sigma_{xi})$ with $\mu_{xi} = (0, 0, \ldots, 0)'$ and $\Sigma_{xi} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$

y_0 is the average from the historical data, $y_0 = \mu_0 = (0, 0, \ldots, 0)'$

p is quality characteristic variables in pxp diagonal matrix, p = 2,3, and 4

Λ is diagonal smoothing parameter matrix equal to diag($\lambda_1, \lambda_2, \ldots, \lambda_p$) with $0< \lambda_j \leq 1$, j =1, 2,…,p and λ= 0.05, 0.1, 0.2, 0.3, 0.35, 0.4, 0.5 and 0.8
I is identity matrix.

The modified equation of SSRM is written by

$$Q(y_i) = \Lambda Q(x_i) + (I-\Lambda)Q(y_{i-1}) \tag{7}$$

where $Q(y_i)$ is the i^{th} SSRM
$Q(y_0)$ is the average from the historical data, $Q(y_0) = (0, 0, \ldots, 0)'$
$Q(x_i)$ is the i^{th} Signed-Rank of x_i

$$\Sigma_{Q(yi)} = (\lambda/2\text{-}\lambda)(1\text{-}(1\text{-}\lambda)^{2i})\Sigma_{Q(xi)} \tag{8}$$

where $\Sigma_{Q(yi)}$ is the exact variance-covariance matrix of $Q(y_i)$
$\Sigma_{Q(xi)}$ is the variance-covariance matrix of $Q(x_i)$

$$Q_i^{SSRM} = Q(y_i)' \, \Sigma_{Q(yi)}^{-1} \, Q(y_i) \tag{9}$$

where Q_i^{SSRM} is the SSRM statistic
If Q_i^{SSRM} >ucl, then the signal gives an out-of-control.

3.2 SSRdM Control Chart

The Spatial Signed-Rank double Multivariate Exponentially Weighted Moving Average (SSRdM) control chart is modified from Double Multivariate Exponentially Weighted Moving Average (dMEWMA) control chart. The equation of dMEWMA is $z_i = \Lambda y_i + (I-\Lambda)z_{i-1}$, i =1, 2,…,n (Alkahtani and Schaffer, 2012)

where z_i is the i^{th} dMEWMA
z_0 is the average from the historical data, $z_0 = \mu_0 = (0, 0, \ldots, 0)'$

Double calculation of modified equation of SSRM from (7) is written by

$$Q(z_i) = Q(y_i) + (I-\Lambda)Q(z_{i-1}) \tag{10}$$

where $Q(z_i)$ is the i^{th} SSRdM
$Q(z_0)$ is the average from the historical data, $Q(z_0) = (0, 0, \ldots, 0)'$

$$\Sigma_{Q(zi)}=\lambda^4/(1-(1-\lambda)^2)^3(1+(1-\lambda)^2-(i+1)^2(1-\lambda)^{2i}+(2i^2+2i\text{-}1)(1-\lambda)^{2i+2}-i^2(1-\lambda)^{2i+4})\Sigma_{Q(xi)} \tag{11}$$

where $\Sigma_{Q(zi)}$ is the exact variance-covariance matrix of $Q(z_i)$

$$Q_i^{SSRdM} = Q(z_i)' \, \Sigma_{Q(zi)}^{-1} \, Q(z_i) \tag{12}$$

where Q_i^{SSRdM} is the SSRdM statistic
If Q_i^{SSRdM} >ucl, then the signal gives an out-of-control.

4. Average Run Length

The performance of control charts is measured by the ARL = 1/Pr. The ARL is a geometric distributed random variable, which gives the probability (Montgomery 2013). The first occurrence of success which requires n independent trials of each success is called probability, Pr, that any point exceeds the control limits. The geometric distribution is written by

$$\text{Prob}(x = i) = (1 - \text{Pr})^{i-1}\text{Pr}, \;\; \text{Pr} \in (0,1) \tag{13}$$

The expected value, E(x) of a geometrically distributed random variable x is ARL. The variance, Var(x) is equal to $(1 - Pr)/Pr^2$ and the standard error of the mean, SE(x) is equal to $\sqrt{Var(x)/n}$.

In the simulation process, the ARL is calculated by the sum of the number of successive points that in-control before a signal move to out-of-control situation divided by the number of repeating. In this article uses n =15,000 units, be random vectors in a process with symmetric non-normal t-distributions with kurtosis equal to 9 at all p. This can be estimated from preliminary p×n sample data matrix with 5,000 sets repeating as NCSS (2018) presented. The simulations are used to determine UCL for $ARL_0 = 370$ at all λ. All ARL calculations are completed based on Monte Carlo simulations via Matlab software.

The ARL performance of the MEWMA, dMEWMA, SSRM, and SSRdM control charts depend on λ, μ and Σ through the non-centrality parameter $\delta = [(\mu_{shift} - \mu_{target})' \Sigma^{-1} (\mu_{shift} - \mu_{target})]^{1/2}$. The δ is used to measure the multivariate distance from μ_{target} (target mean) to μ_{shift} (shift mean). The $\delta = 0$ is in-control but the δ = 0.1, 0.25, 0.5, 1, 1.5, and 2.5 and the process mean, μ_{shift}, are out-of-control.

5. Research Methodology

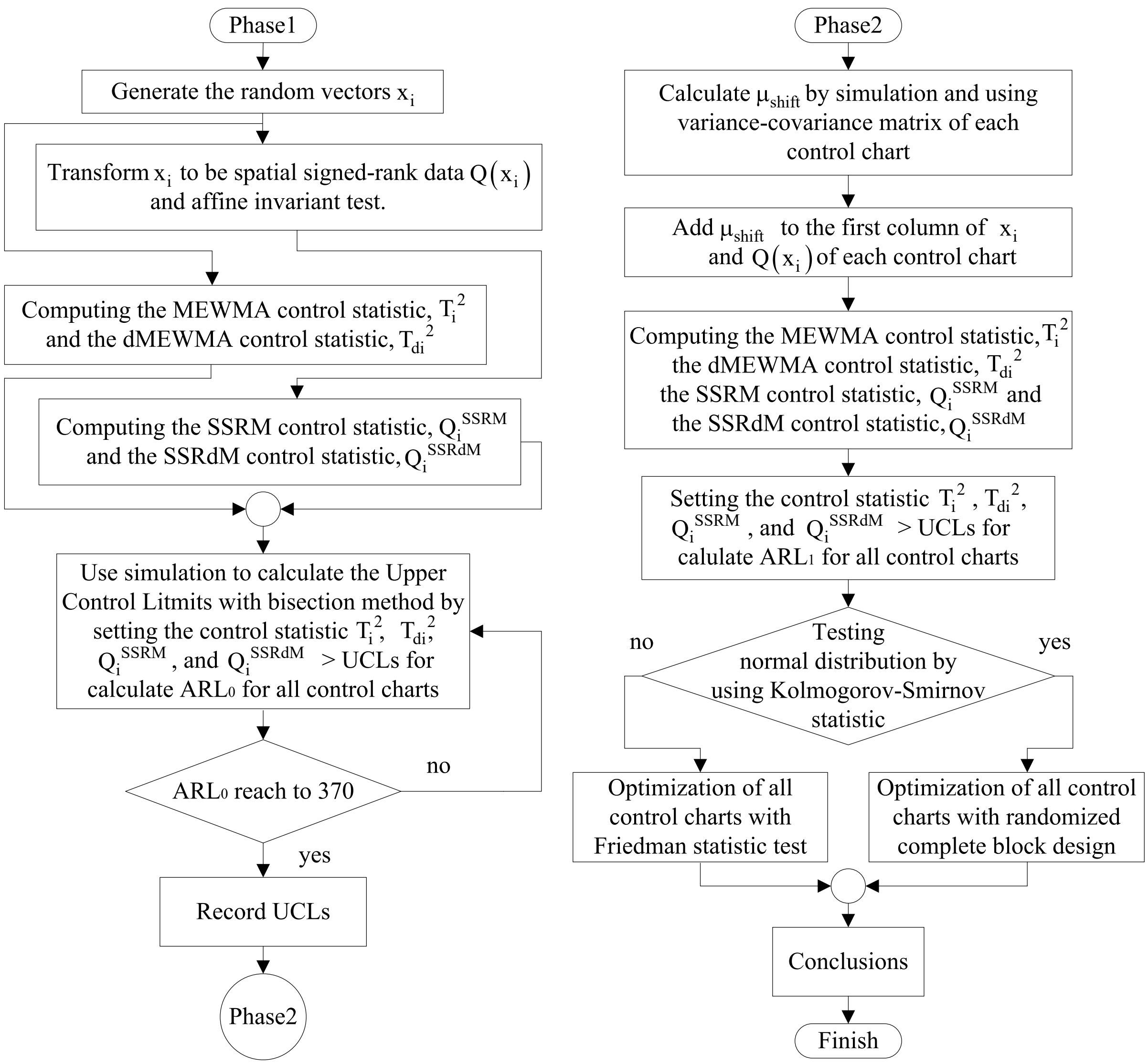
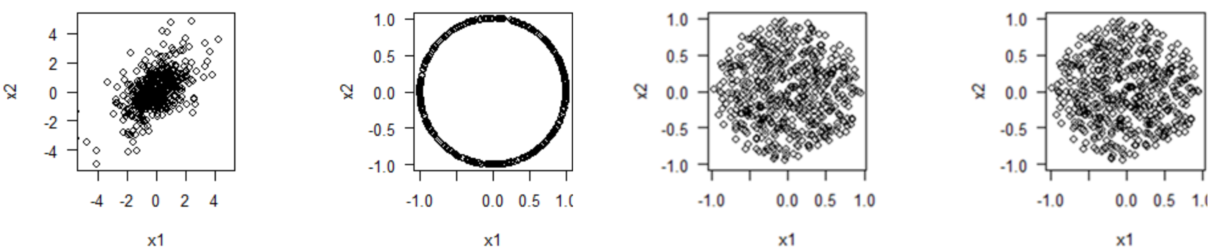


Figure 1. The proposed conceptual model

The Monte Carlo simulation process is conducted in two phases. Phase one calculates the UCLs for the MEWMA, dMEWMA, SSRM, and SSRdM control charts, respectively, for the in-control average run length (ARL_0). Phase two calculates the out-of-control average run length (ARL_1) for monitoring the process mean. Finally, the ARL_1 from phase two simulation from four types of control charts are compared.

6. Results and Discussions

The results of inner standardization to x_i give the shape of the affine invariant and equivariance of the location estimate. The scatter plot of original data transform to spatial signs, spatial ranks and spatial signed-rank as in figure 1.



(a) Original data (b) Spatial signs (c) Spatial ranks (d) Spatial signed-rank

Figure 2. The scatterplots of x_i by 500 samples with inner standardization

The results of ARL_1 comparisons for p (quality characteristic variables), λ (smoothing parameter), and δ (non-centrality parameter) from x_i of the ARL_0 are approximately equal to 370 which is 3σ limits. There is 0.27 percentage of processes outside of the control limit. In Table 1-6, the ARL_1 are rounded-up because in practical ARL_1 is integer number. The SE(x_i) in parentheses are the quality of the mean of the randomized repeatedly that calculated after round-up ARL_1.

The trend of change in the ARL_1 for the MEWMA and dMEWMA are increasing as λ increases. But on the other hand, the ARL_1 for all control charts tend to decrease as δ increases for all p. Consistently with previous studies, the ARL_1 generally tends to increase as λ increases, except for very large values of δ (or large shifts) (Montgomery 2013). For the SSRM and SSRdM control charts are modified with the Multivariate Spatial Signs and Ranks that affect the UCLs of both charts decrease continuously. The UCLs of the SSRM chart decrease quickly at $\lambda \geq 0.35$. These make the charts more effective in detecting waste at the λ more increase. The ARL_1 for the SSRM tends to decrease when $\lambda \geq 0.35$ for all p and the SSRdM tends to decrease when $\lambda \geq 0.4$ at p = 2 and 3, and $\lambda = 0.8$ at p = 4. The decreasing of UCLs affect the ARL_1 decreases, it indicates that both SSRM and SSRdM are more effective in detecting more waste when data are standardized by Spatial Signs and Ranks.

The values of ARL_1 for all control charts tend to increase when p is increased. The increasing of p affect the value of $\sum^{-1}$ increase then values of UCLs increase. These effects reduce the sensitivity of the detection of waste when p is increased. The results consistent to the previously studied as shown in “introduction to statistical quality control” book (Montgomery 2013).

The p-value of Kolmogorov–Smirnov test equal to 0.00 that has significant differences, the ARL_1 are not normally distributed. Therefore, the Friedman test is chosen to test the performance of control charts which the p-value of Friedman test equal to 0.00 that has significant differences. The star-sign in the blanket, [*], indicates the rank of the means of ARL_1 of the control chart and shows that control chart has the most performance.

Table 1. ARL_1 comparisons of λ = 0.05, 0.1, 0.2, and 0.3 and δ from x_i of $ARL_0 \cong 370$ at p = 2

δ	λ															
	0.05				0.1				0.2				0.3			
	UCL															
	9.73	6.20	8.76	5.91	11.88	7.76	9.49	7.28	15.29	9.95	9.49	8.72	18.41	11.81	9.01	9.37
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0.00	370 (5.23)	370 (5.23)	370 (5.23)	371 (5.24)	371 (5.24)	371 (5.24)	371 (5.24)	371 (5.24)	371 (5.24)	371 (5.24)	371 (5.24)	370 (5.23)	370 (5.23)	370 (5.23)	370 (5.23)	371 (5.24)
0.10	258 (3.64) [2.46]	220 (3.10) [2.72]	236 (3.33) [2.30*]	207 (2.92) [2.53]	304 (4.29) [2.49]	255 (3.60) [2.69]	260 (3.67) [2.26*]	244 (3.44) [2.55]	346 (4.89) [2.50]	306 (4.32) [2.74]	271 (3.83) [2.25*]	274 (3.87) [2.52]	362 (5.11) [2.52]	333 (4.70) [2.73]	281 (3.97) [2.28*]	286 (4.04) [2.47]
0.25	98 (1.38) [2.77]	71 (1.00) [2.59]	81 (1.14) [2.25*]	68 (0.95) [2.39]	154 (2.17) [2.96]	94 (1.32) [2.60]	99 (1.39) [2.17*]	85 (1.19) [2.28]	255 (3.60) [3.01]	146 (2.06) [2.75]	111 (1.56) [1.99*]	110 (1.55) [2.25]	312 (4.41) [3.00]	204 (2.88) [2.85]	117 (1.65) [1.92*]	126 (1.77) [2.23]
0.50	29 (0.40) [2.83]	23 (0.32) [2.53]	25 (0.35) [2.29*]	22 (0.30) [2.36]	45 (0.63) [3.20]	27 (0.37) [2.46]	29 (0.40) [2.11*]	26 (0.36) [2.24]	106 (1.49) [3.48]	41 (0.57) [2.63]	33 (0.46) [1.83*]	31 (0.43) [2.06]	193 (2.72) [3.54]	67 (0.94) [2.82]	35 (0.49) [1.66*]	37 (0.52) [1.98]
1.00	8 (0.11) [2.96]	7 (0.09) [2.40]	8 (0.11) [2.46]	7 (0.09) [2.19*]	11 (0.15) [3.25]	8 (0.11) [2.44]	8 (0.11) [2.15*]	8 (0.11) [2.17]	19 (0.26) [3.64]	10 (0.13) [2.54]	9 (0.12) [1.78*]	8 (0.11) [2.04]	44 (0.62) [3.78]	12 (0.16) [2.73]	9 (0.12) [1.58*]	9 (0.12) [1.91]
1.50	4 (0.05) [3.04]	3 (0.03) [2.26]	4 (0.05) [2.58]	3 (0.03) [2.13*]	5 (0.06) [3.23]	4 (0.05) [2.36]	4 (0.05) [2.28]	4 (0.05) [2.14*]	7 (0.09) [3.61]	4 (0.05) [2.52]	4 (0.05) [1.83*]	4 (0.05) [2.05]	12 (0.16) [3.80]	5 (0.06) [2.67]	4 (0.05) [1.60*]	4 (0.05) [1.93]
2.50	2 (0.02) [2.95]	1 (0.00) [2.17*]	1 (0.00) [2.68]	1 (0.00) [2.21]	2 (0.02) [3.09]	1 (0.00) [2.24]	1 (0.00) [2.49]	1 (0.00) [2.17*]	2 (0.02) [3.39]	2 (0.02) [2.40]	1 (0.00) [2.12]	1 (0.00) [2.10*]	3 (0.03) [3.63]	2 (0.02) [2.53]	1 (0.00) [1.85*]	2 (0.02) [1.99]

(i) MEWMA control chart
(ii) dMEWMA control chart
(iii) SSRM control chart
(iv) SSRdM control chart

Table 2. ARL_1 comparisons of $\lambda = 0.35, 0.4, 0.5$, and 0.8 and δ from x_i of $ARL_0 \cong 370$ at $p = 2$

δ	λ															
	0.35				0.4				0.5				0.8			
	UCL															
	19.76	12.73	8.69	9.49	21.07	13.66	8.36	9.51	23.44	15.71	7.67	9.29	28.24	25.74	5.68	6.81
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0.00	371 (5.24)	370 (5.23)	370 (5.23)	370 (5.23)	370 (5.23)	370 (5.23)	371 (5.24)	370 (5.23)	370 (5.23)	371 (5.24)	370 (5.23)	371 (5.24)	371 (5.24)	371 (5.24)	371 (5.24)	370 (5.23)
0.10	362 (5.11) [2.52]	341 (4.82) [2.73]	279 (3.94) [2.28*]	289 (4.08) [2.48]	364 (5.14) [2.53]	350 (4.94) [2.72]	280 (3.95) [2.29*]	288 (4.07) [2.45]	364 (5.14) [2.59]	355 (5.01) [2.66]	275 (3.88) [2.30*]	291 (4.11) [2.45]	369 (5.21) [2.75]	369 (5.21) [2.74]	216 (3.05) [2.14*]	253 (3.57) [2.37]
0.25	324 (4.57) [2.99]	232 (3.27) [2.89]	116 (1.63) [1.89*]	131 (1.85) [2.23]	332 (4.69) [2.99]	258 (3.64) [2.93]	114 (1.61) [1.87*]	132 (1.86) [2.21]	346 (4.89) [3.02]	295 (4.16) [2.98]	108 (1.52) [1.82*]	130 (1.83) [2.17]	364 (5.14) [3.18]	356 (5.03) [3.16]	67 (0.94) [1.63*]	95 (1.34) [2.03]
0.50	224 (3.16) [3.52]	86 (1.21) [2.91]	35 (0.49) [1.60*]	39 (0.54) [1.97]	251 (3.54) [3.52]	109 (1.53) [2.97]	34 (0.47) [1.57*]	40 (0.56) [1.95]	286 (4.04) [3.48]	163 (2.30) [3.09]	32 (0.45) [1.51*]	40 (0.56) [1.92]	336 (4.74) [3.41]	319 (4.50) [3.36]	19 (0.26) [1.40*]	28 (0.39) [1.83]
1.00	66 (0.93) [3.81]	15 (0.20) [2.81]	8 (0.11) [1.50*]	9 (0.12) [1.88]	91 (1.28) [3.82]	19 (0.26) [2.88]	8 (0.11) [1.45*]	9 (0.12) [1.85]	142 (2.00) [3.82]	33 (0.46) [2.98]	8 (0.11) [1.39*]	9 (0.12) [1.81]	256 (3.61) [3.57]	207 (2.92) [3.37]	5 (0.06) [1.38*]	7 (0.09) [1.69]
1.50	17 (0.23) [3.84]	6 (0.08) [2.76]	4 (0.05) [1.52*]	4 (0.05) [1.88]	25 (0.35) [3.87]	6 (0.08) [2.83]	3 (0.03) [1.47*]	4 (0.05) [1.84]	51 (0.71) [3.89]	9 (0.12) [2.93]	3 (0.03) [1.41*]	4 (0.05) [1.77]	162 (2.28) [3.66]	102 (1.44) [3.29]	2 (0.02) [1.43*]	3 (0.03) [1.62]
2.50	4 (0.05) [3.72]	2 (0.02) [2.60]	1 (0.00) [1.75*]	2 (0.02) [1.94]	4 (0.05) [3.78]	2 (0.02) [2.67]	1 (0.00) [1.67*]	2 (0.02) [1.88]	7 (0.09) [3.86]	2 (0.02) [2.80]	1 (0.00) [1.57*]	1 (0.00) [1.78]	43 (0.60) [3.79]	16 (0.22) [3.15]	1 (0.00) [1.48*]	1 (0.00) [1.57]

(i) MEWMA control chart
(ii) dMEWMA control chart
(iii) SSRM control chart
(iv) SSRdM control chart

Table 3. ARL_1 comparisons of λ = 0.05, 0.1, 0.2, and 0.3 and δ from x_i of $ARL_0 \cong 370$ at p = 3

δ	λ															
	0.05				0.1				0.2				0.3			
	UCL															
	12.30	8.08	10.73	7.63	15.05	10.01	11.54	9.19	19.88	12.68	11.59	10.67	24.11	15.07	11.09	11.37
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0.00	371 (5.24)	371 (5.24)	371 (5.24)	370 (5.23)	370 (5.23)	371 (5.24)	371 (5.24)	370 (5.23)	371 (5.24)	370 (5.23)	371 (5.24)	370 (5.23)	371 (5.24)	371 (5.24)	371 (5.24)	370 (5.23)
0.10	283 (4.00) [2.50]	234 (3.30) [2.70]	248 (3.50) [2.28*]	222 (3.13) [2.53]	327 (4.62) [2.48]	282 (3.98) [2.75]	279 (3.94) [2.27*]	258 (3.64) [2.51]	357 (5.04) [2.46]	322 (4.55) [2.74]	291 (4.11) [2.29*]	285 (4.02) [2.51]	364 (5.14) [2.45]	342 (4.83) [2.72]	302 (4.26) [2.33*]	300 (4.24) [2.50]
0.25	115 (1.62) [2.83]	79 (1.11) [2.57]	89 (1.25) [2.26*]	74 (1.04) [2.34]	193 (2.72) [2.97]	112 (1.58) [2.62]	114 (1.61) [2.18*]	95 (1.34) [2.24]	295 (4.16) [2.95]	183 (2.58) [2.78]	142 (2.00) [2.06*]	129 (1.82) [2.20]	334 (4.72) [2.86]	247 (3.49) [2.87]	152 (2.14) [2.06*]	148 (2.09) [2.21]
0.50	34 (0.47) [2.96]	26 (0.36) [2.57]	28 (0.39) [2.20*]	25 (0.35) [2.27]	58 (0.81) [3.28]	32 (0.45) [2.55]	33 (0.46) [2.05*]	29 (0.40) [2.13]	161 (2.27) [3.49]	52 (0.73) [2.70]	40 (0.56) [1.87*]	35 (0.49) [1.94]	242 (3.42) [3.46]	94 (1.32) [2.89]	44 (0.62) [1.73*]	43 (0.60) [1.91]
1.00	10 (0.13) [3.05]	8 (0.11) [2.54]	8 (0.11) [2.30]	7 (0.09) [2.11*]	13 (0.18) [3.35]	9 (0.12) [2.57]	9 (0.12) [2.01*]	8 (0.11) [2.07]	29 (0.40) [3.71]	11 (0.15) [2.64]	10 (0.13) [1.75*]	9 (0.12) [1.90]	79 (1.11) [3.81]	16 (0.22) [2.81]	10 (0.13) [1.59*]	10 (0.13) [1.80]
1.50	5 (0.06) [3.15]	4 (0.05) [2.39]	4 (0.05) [2.43]	3 (0.03) [2.03*]	6 (0.08) [3.35]	4 (0.05) [2.51]	4 (0.05) [2.12]	4 (0.05) [2.02*]	10 (0.13) [3.72]	5 (0.06) [2.64]	4 (0.05) [1.75*]	4 (0.05) [1.90]	21 (0.29) [3.85]	6 (0.08) [2.77]	4 (0.05) [1.57*]	4 (0.05) [1.81]
2.50	2 (0.02) [3.09]	1 (0.00) [2.24]	2 (0.05) [2.59]	1 (0.00) [2.08*]	2 (0.02) [3.26]	2 (0.02) [2.35]	2 (0.02) [2.36]	1 (0.00) [2.03*]	3 (0.03) [3.59]	2 (0.02) [2.51]	2 (0.02) [1.97]	2 (0.02) [1.94*]	4 (0.05) [3.78]	2 (0.02) [2.65]	2 (0.02) [1.72*]	2 (0.02) [1.85]

(i) MEWMA control chart
(ii) dMEWMA control chart
(iii) SSRM control chart
(iv) SSRdM control chart

Table 4. ARL_1 comparisons of $\lambda = 0.35, 0.4, 0.5$, and 0.8 and δ from x_i of $ARL_0 \cong 370$ at $p = 3$

δ	λ															
	0.35				0.4				0.5				0.8			
	UCL															
	26.03	16.31	10.77	11.56	27.89	17.50	10.42	11.58	31.15	20.27	9.69	11.38	37.77	34.31	7.49	8.77
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0.00	370 (5.23)	371 (5.24)	371 (5.24)	371 (5.24)	370 (5.23)	371 (5.24)	371 (5.24)	370 (5.23)	371 (5.24)	371 (5.24)	370 (5.23)	371 (5.24)	371 (5.24)	370 (5.23)	371 (5.24)	370 (5.23)
0.10	362 (5.11) [2.45]	352 (4.97) [2.71]	301 (4.25) [2.34*]	308 (4.35) [2.51]	366 (5.17) [2.49]	357 (5.04) [2.67]	300 (4.24) [2.36*]	305 (4.31) [2.48]	369 (5.21) [2.55]	359 (5.07) [2.61]	300 (4.24) [2.38*]	305 (4.31) [2.46]	369 (5.21) [2.64]	369 (5.21) [2.64]	267 (3.77) [2.28*]	299 (4.22) [2.44]
0.25	341 (4.82) [2.84]	273 (3.85) [2.90]	154 (2.17) [2.05*]	158 (2.23) [2.22]	344 (4.86) [2.85]	289 (4.08) [2.89]	154 (2.17) [2.05*]	160 (2.26) [2.21]	354 (5.00) [2.89]	316 (4.46) [2.86]	152 (2.14) [2.03*]	168 (2.37) [2.22]	363 (5.13) [3.02]	361 (5.10) [3.02]	106 (1.49) [1.79*]	142 (2.00) [2.17]
0.50	268 (3.78) [3.41]	125 (1.76) [2.97]	45 (0.63) [1.70*]	46 (0.64) [1.93]	285 (4.02) [3.39]	154 (2.17) [3.01]	46 (0.64) [1.68*]	48 (0.67) [1.92]	312 (4.41) [3.37]	210 (2.96) [3.09]	44 (0.62) [1.63*]	52 (0.73) [1.91]	346 (4.89) [3.36]	332 (4.69) [3.33]	29 (0.40) [1.47*]	42 (0.59) [1.85]
1.00	112 (1.58) [3.82]	21 (0.29) [2.88]	10 (0.13) [1.54*]	10 (0.13) [1.77]	146 (2.06) [3.82]	28 (0.39) [2.93]	10 (0.13) [1.50*]	10 (0.13) [1.75]	201 (2.84) [3.79]	56 (0.78) [3.02]	9 (0.12) [1.45*]	11 (0.15) [1.74]	290 (4.09) [3.52]	255 (3.60) [3.40]	7 (0.09) [1.39*]	9 (0.12) [1.69]
1.50	32 (0.45) [3.87]	7 (0.09) [2.84]	4 (0.05) [1.50*]	4 (0.05) [1.79]	50 (0.70) [3.89]	8 (0.11) [2.89]	4 (0.05) [1.46*]	4 (0.05) [1.75]	97 (1.36) [3.89]	14 (0.19) [2.97]	4 (0.05) [1.42*]	4 (0.05) [1.72]	218 (3.08) [3.61]	162 (2.28) [3.34]	3 (0.03) [1.42*]	4 (0.05) [1.62]
2.50	5 (0.06) [3.83]	2 (0.02) [2.72]	2 (0.02) [1.64*]	2 (0.02) [1.82]	7 (0.09) [3.87]	3 (0.03) [2.78]	1 (0.00) [1.57*]	2 (0.02) [1.78]	13 (0.18) [3.91]	3 (0.03) [2.89]	1 (0.00) [1.49*]	2 (0.02) [1.72]	87 (1.22) [3.76]	39 (0.54) [3.20]	1 (0.00) [1.46*]	1 (0.00) [1.57]

(i) MEWMA control chart
(ii) dMEWMA control chart
(iii) SSRM control chart
(iv) SSRdM control chart

Table 5. ARL_1 comparisons of λ = 0.05, 0.1, 0.2, and 0.3 and δ from x_i of $ARL_0 \cong 370$ at p = 4

δ	λ															
	0.05				0.1				0.2				0.3			
	UCL															
	14.62	9.84	12.63	9.25	18.10	11.95	13.50	10.94	23.94	15.15	13.59	12.53	29.43	18.05	13.13	13.29
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0.00	371 (5.24)	370 (5.23)	370 (5.23)	370 (5.23)	370 (5.23)	370 (5.23)	371 (5.24)	370 (5.23)	371 (5.24)	370 (5.23)	370 (5.23)	371 (5.24)	371 (5.24)	371 (5.24)	371 (5.24)	371 (5.24)
0.10	293 (4.14) [2.47]	250 (3.53) [2.71]	260 (3.67) [2.29*]	237 (3.34) [2.54]	337 (4.76) [2.47]	288 (4.07) [2.72]	287 (4.05) [2.27*]	272 (3.84) [2.54]	358 (5.06) [2.44]	329 (4.65) [2.73]	309 (4.36) [2.32*]	297 (4.19) [2.51]	364 (5.14) [2.40]	351 (4.96) [2.72]	326 (4.60) [2.39*]	306 (4.32) [2.49]
0.25	130 (1.83) [2.83]	86 (1.21) [2.59]	98 (1.38) [2.24*]	82 (1.15) [2.34]	220 (3.10) [2.97]	123 (1.73) [2.62]	126 (1.77) [2.17*]	105 (1.48) [2.24]	310 (4.38) [2.88]	206 (2.91) [2.81]	157 (2.21) [2.13*]	136 (1.92) [2.18]	341 (4.82) [2.79]	268 (3.78) [2.88]	179 (2.52) [2.14*]	161 (2.27) [2.19]
0.50	39 (0.54) [2.98]	28 (0.39) [2.57]	31 (0.43) [2.21*]	27 (0.37) [2.24]	74 (1.04) [3.34]	36 (0.50) [2.52]	37 (0.52) [2.07*]	32 (0.45) [2.08]	194 (2.74) [3.48]	64 (0.90) [2.74]	47 (0.66) [1.88*]	40 (0.56) [1.90]	272 (3.84) [3.40]	119 (1.68) [2.92]	54 (0.76) [1.80*]	49 (0.69) [1.88]
1.00	11 (0.15) [3.09]	9 (0.12) [2.57]	9 (0.12) [2.24]	8 (0.11) [2.11*]	15 (0.20) [3.44]	10 (0.13) [2.56]	10 (0.13) [1.96*]	9 (0.12) [2.05]	42 (0.59) [3.75]	13 (0.18) [2.69]	11 (0.15) [1.73*]	10 (0.13) [1.84]	115 (1.62) [3.82]	21 (0.29) [2.84]	11 (0.15) [1.60*]	11 (0.15) [1.74]
1.50	5 (0.06) [3.17]	4 (0.05) [2.45]	4 (0.05) [2.39]	4 (0.05) [2.00*]	7 (0.09) [3.46]	5 (0.06) [2.51]	5 (0.06) [2.06]	4 (0.05) [1.98*]	12 (0.16) [3.77]	6 (0.08) [2.67]	5 (0.06) [1.72*]	5 (0.06) [1.84]	32 (0.45) [3.88]	7 (0.09) [2.82]	5 (0.06) [1.57*]	5 (0.06) [1.74]
2.50	2 (0.02) [3.15]	1 (0.00) [2.29]	2 (0.02) [2.56]	1 (0.00) [2.01*]	3 (0.03) [3.37]	2 (0.02) [2.36]	2 (0.02) [2.30]	2 (0.02) [1.98*]	4 (0.05) [3.69]	2 (0.02) [2.55]	2 (0.02) [1.90]	2 (0.02) [1.86*]	5 (0.06) [3.84]	3 (0.03) [2.71]	2 (0.02) [1.67*]	2 (0.02) [1.78]

(i) MEWMA control chart
(ii) dMEWMA control chart
(iii) SSRM control chart
(iv) SSRdM control chart

Table 6. ARL_1 comparisons of λ = 0.35, 0.4, 0.5, and 0.8 and δ from x_i of $ARL_0 \cong 370$ at p = 4

δ	λ															
	0.35				0.4				0.5				0.8			
	UCL															
	31.96	19.46	12.78	13.47	34.43	21.11	12.41	13.55	38.91	24.50	11.63	13.41	47.12	42.79	9.31	10.69
	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)	(i)	(ii)	(iii)	(iv)
0.00	370 (5.23)	371 (5.24)	370 (5.23)	371 (5.24)	370 (5.23)	370 (5.23)	371 (5.24)	371 (5.24)	370 (5.23)	371 (5.24)	370 (5.23)	370 (5.23)	371 (5.24)	370 (5.23)	370 (5.23)	370 (5.23)
0.10	365 (5.15) [2.43]	352 (4.97) [2.68]	325 (4.59) [2.40*]	313 (4.42) [2.49]	369 (5.21) [2.45]	361 (5.10) [2.67]	322 (4.55) [2.38*]	323 (4.56) [2.49]	372 (5.25) [2.54]	364 (5.14) [2.58]	321 (4.53) [2.39*]	326 (4.60) [2.49]	370 (5.23) [2.60]	370 (5.23) [2.61]	298 (4.21) [2.33*]	325 (4.59) [2.46]
0.25	352 (4.97) [2.80]	286 (4.04) [2.88]	182 (2.57) [2.13*]	169 (2.38) [2.20]	355 (5.01) [2.79]	313 (4.42) [2.88]	182 (2.57) [2.10*]	177 (2.50) [2.22]	366 (5.17) [2.85]	336 (4.74) [2.83]	179 (2.52) [2.08*]	193 (2.72) [2.25]	364 (5.14) [2.93]	365 (5.15) [2.94]	143 (2.02) [1.93*]	175 (2.47) [2.21]
0.50	297 (4.19) [3.38]	153 (2.16) [2.99]	57 (0.80) [1.77*]	53 (0.74) [1.87]	314 (4.43) [3.35]	189 (2.67) [3.04]	58 (0.81) [1.74*]	56 (0.78) [1.87]	338 (4.77) [3.33]	248 (3.50) [3.08]	58 (0.81) [1.70*]	63 (0.88) [1.89]	352 (4.97) [3.30]	349 (4.93) [3.30]	42 (0.59) [1.53*]	57 (0.80) [1.88]
1.00	157 (2.21) [3.82]	28 (0.39) [2.90]	12 (0.16) [1.57*]	11 (0.15) [1.72]	192 (2.71) [3.80]	40 (0.56) [2.95]	12 (0.16) [1.53*]	12 (0.16) [1.71]	250 (3.53) [3.76]	82 (1.15) [3.05]	12 (0.16) [1.48*]	13 (0.18) [1.70]	312 (4.41) [3.50]	288 (4.07) [3.41]	9 (0.12) [1.41*]	11 (0.15) [1.69]
1.50	53 (0.74) [3.90]	9 (0.12) [2.87]	5 (0.06) [1.52*]	5 (0.06) [1.72]	83 (1.17) [3.91]	11 (0.15) [2.91]	5 (0.06) [1.48*]	5 (0.06) [1.70]	148 (2.09) [3.90]	21 (0.29) [2.98]	5 (0.06) [1.44*]	5 (0.06) [1.69]	256 (3.61) [3.57]	210 (2.96) [3.39]	4 (0.05) [1.42*]	4 (0.05) [1.61]
2.50	7 (0.09) [3.88]	3 (0.03) [2.78]	2 (0.02) [1.59*]	2 (0.02) [1.75]	10 (0.13) [3.91]	3 (0.03) [2.84]	2 (0.02) [1.53*]	2 (0.02) [1.72]	26 (0.36) [3.94]	4 (0.05) [2.92]	2 (0.02) [1.47*]	2 (0.02) [1.68]	131 (1.85) [3.72]	71 (1.00) [3.26]	1 (0.00) [1.46*]	1 (0.00) [1.57]

(i) MEWMA control chart
(ii) dMEWMA control chart
(iii) SSRM control chart
(iv) SSRdM control chart

7. Conclusions

Modified Multivariate Control Chart Using Spatial Signs and Ranks for Monitoring Process Mean of symmetric t-distribution data is presented base on the ARL_1 property. The Friedman test is used to compare the means between MEWMA, dMEWMA, SSRM, and SSRdM control charts and to select the highest performance of control charts.

The SSRdM shows better performance at small $0.05 \leq \lambda \leq 0.2$ for all p. When consider at small $\lambda = 0.05$, The SSRdM can detect in moderate shift ($\delta = 1$) and some large shifts ($\delta = 1.5$) and underperform when λ increases, $\lambda = 0.1$ can detect in large shifts ($\delta \geq 1.5$), at $\lambda = 0.2$ can detect in some large shifts ($\delta = 2.5$) and at $0.3 \leq \lambda \leq 0.8$ can detect underperform SSRM. The SSRdM will underperform when λ increases. It indicates that the SSRdM interest in the resolution of data with both historical and current.

The SSRM shows better performance than others for all p. When consider at small $\lambda = 0.05$, The SSRM can detect in small shifts ($\delta \leq 0.5$) and outperform when λ increases, at $\lambda = 0.1$ can detect better in small shifts ($\delta \leq 0.5$) and moderate shift ($\delta = 1$), at $\lambda = 0.2$ can detect better in small shifts ($\delta \leq 0.5$), moderate shift ($\delta = 1$) and some large shifts ($\delta = 1.5$) and at $0.3 \leq \lambda \leq 0.8$ can detect better in all small shifts ($\delta \leq 0.5$), moderate shift ($\delta = 1$) and large shifts ($\delta \geq 1.5$) in the process mean vector. The SSRM will have a better performance when λ increases. It indicates that the SSRM not interest in the resolution of data.

In the industrial world, the data are not a normal distribution and Baxley (1990) reported, the results for a simulated industrial process requiring a larger λ ($\lambda = 0.35$). The results of this study show that the SSRM is superior for detecting small, moderate and large shifts with $\lambda \geq 0.3$ at all of p. And the SSRM is superior for detecting small shifts ($\delta \leq 0.5$) for all λ and p, thus the SSRM is the more modern appropriate alternative for process mean monitoring in industrial. It is suitable to be used in the industry because most of the industry has the current data.

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